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**The Assam Royal Global University, Guwahati**

**Royal School of Applied & Pure Sciences**

**B.Sc. (H) Mathematics, 2<sup>nd</sup> Semester**

**Semester End Examination, June 2023**

**Course Title: Analytical Geometry**

**Course Code: MAT012C202**

Time: 3 Hours

Maximum Marks: 70

**Note: Attempt all questions as per instructions given.**

*The figures in the right-hand margin indicate marks.*

**SECTION – A**

**1. Attempt all questions:**

**2 x 8 = 16**

- Transform parallel axes to the point  $(a, b)$  of equation  $\frac{x}{a} + \frac{y}{b} = 2$ .
- Prove that the equation  $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$  represents a pair of straight lines.
- Find the centre of the conic given by  $3x^2 - 8xy + 7y^2 - 4x + 2y - 7 = 0$ .
- Prove that equation of the tangent to the conic  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  at the origin  $(0,0)$  is  $gx + fy + c = 0$ .
- If the axes are rectangular, find the distance of the plane  $6x - 3y + 2z - 14 = 0$  from the origin.
- Find the angle between the planes  $x - y + z = 1$  and  $2x + y + 3z = 4$ .
- Find the radius and centre of the sphere  $x^2 + y^2 + z^2 - 2x + 4y - 6z = 2$ .
- Find the equation of the sphere whose centre is  $(2, -3, 4)$  and radius 5.

**SECTION – B**

**2. Attempt any two of the following questions:**

**6 x 2 = 12**

- Prove that the orthogonal transformation without the change of origin, the quantity  $(g^2 + f^2)$  in the equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ , is an invariant.
  - If by any change of axes without the change of origin, the expression  $ax^2 + 2hxy + by^2 = 0$  becomes  $ax'^2 + 2h'x'y' + by'^2 = 0$ , then prove that  $a + b = a' + b'$ .
- Show that the angle between the pair of straight lines represented by the equation  $ax^2 + 2hxy + by^2 = 0$  is  $\tan^{-1} \left( \frac{2\sqrt{h^2 - ab}}{a+b} \right)$ .
- Prove that the general equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents two parallel straight lines if  $h^2 = ab$  and  $bg^2 = af^2$ .

**3. Attempt any four of the following questions:**

7 x 2=14

- a. i. Find the equation of the tangent to the conic  $4x^2 + 3xy + 2y^2 - 3x + 5y + 7 = 0$  at the point  $(1, -2)$ .
- ii. Find the equation of the chord of contact  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ .
- b. i. Find the equation of the polar of the point  $(2,3)$  w.r.t. the conic  $x^2 + 3xy + 4y^2 - 5x + 3 = 0$ .
- ii. Prove that the points  $(1,2)$  and  $(-2,3)$  are conjugate w.r.t. the conic  $2x^2 + 6xy + y^2 + 4x - 2y + 8 = 0$
- c. i. Deduce the standard equation of the parabola.
- ii. Find the equation of the parabola whose focus is  $(-1, 1)$  and whose directrix is  $x + y + 1 = 0$ .

**4. Attempt any two of the following questions:**

7 x 2=14

- a) i. Find the equation of the plane  $Ax + By + Cz + D = 0$  in the intercept form.
- ii. A plane meets the coordinate axes at  $A, B, C$  such that centroid of the  $\Delta ABC$  is the point  $(a, b, c)$ ; show that the equation to the plane is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ .
- b) Prove that the four points whose coordinates are  $(0, -1, 0), (2,1, -1), (1,1,1)$  and  $(3,3,0)$  are coplanar and find the equation of the plane containing them.
- c) Find the equation of the plane through the three points  $(0,1,1), (1,1,2), (-1,2, -2)$ .

**5. Attempt any two of the following questions:**

7 x 2=14

- a) (i) Find the equation of the sphere through the four points.
- (ii) Find the equation of the sphere which passes through origin and  $(a, 0, 0), (0, b, 0)$  and  $(0, 0, c)$ .
- b) Find the equation of the sphere passing through the points  $(0, 0, 0), (0, 1, -1), (-1, 2, 0)$  and  $(1, 2, 3)$ .
- c) A plane passes through a fixed point  $(a, b, c)$  and cuts the axes in  $A, B, C$ . Show that the locus of the centre of the sphere  $OABC$  is  $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$

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