Total No. of printed pages =03

SUBJECT CODE: MAT052104

Roll No. of candidate

2017
End Semester BCA Examination
1st Semester
MATHEMATICS-I

Full Marks- 70 Pass Marks: 21 Time- 3 hours

The figures in the margin indicate full marks for the questions.

PART – A

Q1. Answer all questions: $[\mathbf{16 \times 1 = 16}]$

a) Give an example of a bounded above sequence.
b) What are the limit points of the sequence $\{(-1)^n\}$?
c) Under what condition a monotonic sequence becomes convergent?
d) What is alternating series?
e) Define a group.
f) Give an example of a ring.
g) State Lagrange's theorem on the order of a subgroup of a finite group.
h) Give an example of a field.
i) If α, β, γ are roots of the equation $x^3 - 7x^2 + 36 = 0$, then find the value of $\alpha + \beta + \gamma$.
j) If two roots of an cubic equation are 2 and $1 + i$, then find the third root of the equation.
k) State Descarte's rule of sign for roots of an equation.
l) Write the necessary condition for maxima and minima of a function $f(x)$ at point $x = c$.
m) Find modulus of the complex number $= \frac{1+i}{1-i}$.
n) Write the complex number $z = \sqrt{3} - i$ in polar form.
o) If $z = cos\theta + isin\theta$, then find $z^3 + \frac{1}{z^3}$.
p) If z is a complex number, then show that $\cos(iz) = \cosh(z)$.

P.T.O.

PART – B

Q2. Answer all questions: **$[4 \times 3.5 = 14]$**

a) Check the convergence of the sequence $\{n + (-1)^n n\}$.
b) Give an example to show that union of two subgroups is not necessarily a subgroup.
c) If α, β, γ are roots of the equation $x^3 - px^2 + qx - r = 0$, then find the value of $\sum \alpha^2$.
d) Evaluate the Gregory series $\left(\frac{2}{3} + \frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^3} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right) - \cdots \infty$.

PART – C
(All questions are compulsory)

Q.3. **$[5 + 5 = 10]$**

a) Show that a sequence cannot converge to more than one limit.
b) Show that the series 1 - $\frac{1}{2^3}$ + $\frac{1}{3^3}$ - … is absolutely convergent.

OR

a) Show that the sequence $\{S_n\}$ where $S_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n}$ is convergent.
b) Test the convergency of $\sum \frac{n^2-1}{n^2+1} x^n, x > 0$, using D' Alembert's ratio test.

Q4. **$[5 + 5 = 10]$**

a) Use De Moivre's theorem to prove that $5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$.
b) Prove that $\frac{(\cos\theta + i\sin\theta)^4}{(i\cos\theta + \sin\theta)^5} = \sin 9\theta - i\cos 9\theta$ and find all the values of $(1 - i\sqrt{3})^{1/4}$.

OR

a) Expand $\sin^3 x$ in power of x.
b) If x, y are real numbers, then show that
   i) $\sinh(x + iy) = \sinh x \cos y + i \cosh x \sin y$
   ii) $\cos(x + y) = \cosh x \cosh y + \sinh x \sinh y$

Q5. **$[5 + 5 = 10]$**

a) If $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, $g = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 1 & 3 \end{pmatrix}$, show that $fg \neq gf$.
b) Prove that every cyclic group is abelian.

OR

a) Show that the mapping $f: R \to R^+$ given by $f(a) = 2^a, a \in R$, where R is the group of real numbers under addition and R^+, the group of positive real numbers under multiplication is a homomorphism.
b) If R is a ring, then show that $a0 = 0a = 0$, for every $a \in R$ and where 0 is the additive identity of the ring R.

P.T.O.

Q6. **[$\mathbf{5+5=10}$]**

a) Solve the cubic equation by Cardon's method $x^3 - 27x + 54 = 0$.

b) Solve the cubic equation $x^3 - 5x^2 - 2x + 24 = 0$, given that the product of two of the roots is 12.

OR

a) Find for what values of x, the expression $x^3 - 9x^2 + 15x - 3$ is maximum and minimum. Also find its maximum and minimum values.

b) Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$.

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