THE ASSAM ROYAL GLOBAL UNIVERSITY
GUWAHATI

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SUBJECT CODE: MAT022102

Roll No. of candidate

2017

End Semester B.Tech. Examination

1st Semester

APPLIED MATHEMATICS-I

Full Marks- 70 Pass Marks- 21 Time- 3 hours

The figures in the margin indicate full marks.

PART – A
(Marks: 16)

1. Answer all questions: **16 x 1 = 16**

a) If $y = e^{2x}$, find y_n.
b) If $z = x^2y^3$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
c) Give the geometrical interpretation of Rolle 's theorem.
d) State the Leibnitz's theorem for n^{th} derivative of product of two functions.
e) If $= x^2y + 5y^3$, then find the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$
f) If $u = xy$, $x = 3t$, $y = t$, find $\frac{du}{dt}$.
g) Evaluate: $\int_0^{\frac{\pi}{2}} sin^4x\, cos^2x dx$.
h) Discuss the symmetry of the curve: $y^2(a^2 + x^2) = x^2(a^2 - x^2)$
i) Find the volume of the solid generated by revolving the line $y = x$ from $x = 0$ to $x = 1$ about the x-axis.
j) State Leibnitz's rule for differentiation under integral sign.
k) Evaluate: $\int_0^1 \int_1^2 (xy) dxdy$.
l) Compute: $\Gamma\left(\frac{3}{2}\right)$
m) Write down the condition of co-planarity of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.
n) If $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{\sqrt{2}}\,\hat{b}$, then find the angle which $\hat{a}$ makes with $\hat{c}$.

o) Find the vector equation of the straight line passing through the point $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and parallel to the line $\hat{\imath} + \hat{\jmath} - \hat{k}$.

p) Show that the vector $\vec{r} = (x + 3y)\hat{\imath} + (y - 3z)\hat{\jmath} + (x - 2z)\hat{k}$ is solenoidal.

PART – B
(Marks: 14)

2. Answer all questions: **3.5 x 4=14**

a) Obtain the reduction formula for $\int_0^{\frac{\pi}{2}} sin^n x dx$.

b) Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1--x^2-y^2}} \frac{1}{\sqrt{1--x^2-y^2-z^2}} dzdydx$

c) Expand $2x^3 + 7x^2 + x - 1$ in powers of $x - 2$.

d) If $(\vec{a}, \vec{b}, \vec{c})$ and $(\vec{\acute{a}}, \vec{\acute{b}}, \vec{\acute{c}})$ are reciprocal triad of vectors, prove that

$$\vec{a} \times \vec{\acute{a}} + \vec{b} \times \vec{\acute{b}} + \vec{c} \times \vec{\acute{c}} = \vec{0} .$$

PART – C
(Marks: 40)

3. Answer the following questions: **5+5=10**

a) Verify Lagrange's mean value theorem for the function
$f(x) = x^2$ in the interval [2, 4].

b) If $y = tan^{-1}x$, then prove that
$(1 + x^2)y_{n+1} + 2nxy_n + n(n - 1)y_{n-1} = 0.$

OR

a) Find the area included between the curve $xy^2 = a^2(a - x)$ and its asymptote.

b) Find the volume of the solid generated by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line

4. Answer the following questions: **5+5=10**

a) Verify Euler's theorem on the homogeneous function
$z = \sin(\frac{x^2+ y^2}{xy}).$

b) If $u = f(y - z,\ z - x,\ x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$

OR

a) Evaluate $\int_0^\infty \frac{e^{-ax}\sin x}{x}dx$ by applying differentiation under integral sign and hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$.

b) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

5. Answer the following questions: 5+5=10

a) Find the extreme value of the function

$$f(x,y)=4x^2+9y^2-8x-12y+4 .$$

b) If $u=xyz,\ \ v=xy+yz+zx,\ \ w=x+y+z,$ compute $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

OR

a) Prove that the surface area of a sphere is $4\pi a^2$, where a is the radius of the sphere.

b) Express the following in terms of Gamma functions:

(i) $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ (ii) $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$

6. Answer the following questions: 5+5=10

a) Find the directional derivative of $\emptyset = x^2yz+4xz^2$ at the point $(1,-2,-1)$ in the direction of $\widehat{2\imath}-\hat{\jmath}-2\hat{k}$.

b) If $\vec{F}=ax\hat{\imath}+by\hat{\jmath}+cz\hat{k}$, a, b, c are constants , show that $\iint_S \vec{F}.\hat{n}\, \mathrm{dS}=\frac{4}{3}\pi(a+b+c),$ where S is the surface of a unit sphere.

OR

a) Find the divergence and curl of the vector

$\vec{v}=(\mathrm{xyz})\hat{\imath}-(3\mathrm{xy}^2)\hat{\jmath}+(\mathrm{xz}^2-\mathrm{y}^2\mathrm{z})\hat{\mathrm{k}}$ at the point $(1,-2,\ 3)$.

b) Verify Green's Theorem in the plane for $\oint(3\mathrm{x}^2-8\mathrm{y}^2)\mathrm{dx}+(4\mathrm{y}-6\mathrm{xy})\mathrm{dy}$, where C is the boundary of the region defined by $\mathrm{x}=0, \mathrm{y}=0$ and $\mathrm{x}+\mathrm{y}=1$.