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SUBJECT CODE : MAT024101

Roll No. of candidate

2017

End Semester M.Tech. (CIVIL) Examination

1st Semester

ADVANCED ENGINEERING MATHEMATICS

Full Marks- 70 Pass Marks- 21 Time- 3 hours

The figures in the margin indicate full marks.

PART – A
(Marks: 16)

1. Answer all questions: 16 x 1 = 16

a) Evaluate: $L^{-1}\left\{\frac{s}{(s+1)^2}\right\}$

b) Show that $x = 0$ is an ordinary point of $(x^2 - 1)y'' + xy' - y = 0$.

c) Write down the condition of hyperbolic partial differential equation.

d) Classify the following partial differential equation: $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

e) What do you mean by Rank of a matrix?

f) Reduce the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 1 & 8 & 6 \end{bmatrix}$ to upper triangular form.

g) Write down the condition for a system of equations to have unique solution.

h) When the system of equations is said to be ill-conditioned?

i) Write the Newton Gregory forward interpolation formula.

j) Find the polynomial $f(x)$ if $f(0) = 1,\ f(1) = 2, f(3) = 1$.

k) For f(x) $= x^2$, find $f(a, b)$.

l) Write the formula for Euler's method for finding the solution of differential equation $\frac{dy}{dx} = f(x, y)$.

m) Define Poisson distribution.

n) If $X \sim B(n,p)$ with mean 3 and variance 2, what is n?

o) If A and B independent events with probabilities $\frac{1}{2}, \frac{1}{6}$ respectively, find probability of neither A nor B.

p) Write one application of t- test.

PART – B
(Marks: 14)

2. Answer all questions: 3.5 x 4=14

a) Solve : $y''(t) + 3y'(t) + 2y(t) = 0,\ y(0) = 0,\ y'(0) = 1$

b) Reduce the matrix $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ to Echelon form and hence find its rank.

c) Using Euler's predictor –corrector method, find $y(1.1)$ correct upto two decimal places: $\frac{dy}{dx} = 3x + y^2,\ \ y(1) = 1.2$

d) Suppose X has a Poisson distribution. If $P(2) = \frac{2}{3}P(1)$, evaluate

(i) $P(X \geq 2)$

(ii) mean and variance

PART – C
(Marks: 40)

3. Answer the following question: 10

a) Find the power series solution in powers of $(x-1)$ of the initial value problem $xy'' + y' + 2y = 0, y(1) = 1, y'(1) = 2.$

OR

b) Solve the Wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ by using the method of separation of variables and hence solve the following problem:
A string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string in the form $y = A \sin\frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = A \sin\frac{\pi x}{l} \cos\frac{\pi c t}{l}$.

4. Answer the following question: **10**

a) Determine for what values of λ and μ the following equations have (i) no solution (ii) a unique solution (iii) infinite number of solutions

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu$$

OR

b) Solve the following system of equations by Cholesky method:

$$x + 2y + 3z = 5, \quad 2x + 8y + 22z = 6, \quad 3x + 22y + 82z = -10$$

5. Answer the following question: **10**

a) Determine the Hermite polynomial of degree 5 which fits the following data

x	$y = \ln(x)$	$y' = \frac{1}{x}$
2.0	0.69315	0.5
2.5	0.91629	0.4000
3.0	1.09861	0.33333

OR

b) Use Runge-Kutta method of fourth order to find $y(0.1)$, $y(0.2)$; given that $\frac{dy}{dx} = y - x, y(0) = 2$.

6. Answer the following question: **10**

a) A survey of 320 families with 5 children each revealed the following distribution:

No. of boys:	5	4	3	2	1	0
No. of girls:	0	1	2	3	4	5
No. of families:	14	56	110	88	40	12

Is this result consistent with the hypothesis that male and female births are equally probable?

OR

b) A random sample of 10 boys had the following I.Q.'s: 70, 120, 110, 101, 88, 83, 95, 98, 107 and 100. Do these data support the assumption of a population mean I.Q. of 160?