

Total No. of printed pages = 3

SUBJECT CODE :MAT024101

Roll No. of candidate

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2017

End Semester M.Tech.(Mechanical Engineering) Examination

1st Semester

ADVANCED ENGINEERING MATHEMATICS

Full Marks- 70

Pass marks:21

Time- 3 hours

The figures in the margin indicate full marks for the questions

PART – A

1. Answer all questions:

[16 × 1 = 16]

- (a) What is the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 7 \\ 0 & 0 & 6 \end{bmatrix}$?
- (b) What do you mean by ill- condition system of linear equations?
- (c) Does a system of homogeneous linear equations inconsistent? Justify your answer.
- (d) Is the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 4 & 6 \end{bmatrix}$ invertible? Justify your answer.
- (e) What is interpolation?
- (f) Find $f(2)$ if $f(0) = 1, f(1) = -1, f(3) = 0$.
- (g) For $f(x) = x$, find $f(a, b, c)$.
- (h) Give the error of Euler’s method of solving differential equation.
- (i) If A and B are independent events with probabilities $\frac{1}{2}$ and $\frac{1}{5}$ respectively, evaluate $P(A\bar{B})$.
- (j) Define Null Hypothesis.
- (k) Write one application of t- test.
- (l) If X follows poisson distribution with mean 2, evaluate $P(X \geq 1)$.
- (m) What is $\Delta^2 y_x$, $y_x = a + bx$, a and b are constants.
- (n) Which value is minimized by least square method?
- (o) Which curve may be taken as the best fit if $\Delta^2 y_x = \text{constant}$?
- (p) Write one important use of fitting a curve for given data set.

PART – B

2. Answer the following:

- (a) Reduce the matrix $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 4 & 5 \\ 5 & 6 & 2 \end{bmatrix}$ to the normal form. [5]
- (b) Using Newton’s divided difference formula find the missing value from the table: [5]

| | | | | | |
|-----|----|----|---|-----|---|
| x | 1 | 2 | 4 | 5 | 6 |
| y | 14 | 15 | 5 | --- | 9 |

- (c) A bag contains 10 gold and 8 silver coins. Two successive drawings of 4 coins are made such that: (i) coins are replaced before the second trial, (ii) the coins are not replaced before the second trial. Find the probability that the first drawing will give 4 gold and second 4 silver coins. [4]

PART – C

3. Answer the following:

- (a) Find the solution of the following system of linear equations by LU decomposition method

$$2x - 3y + 10z = 3, \quad -x + 4y + 2z = 20, \quad 5x + 2y + z = -12 \quad [10]$$

OR

- (b) Solve the following system of linear equations by Gauss elimination method

$$\begin{aligned} 10x - 7y + 3z + 5w &= 6, & -6x + 8y - z - 4w &= 5, \\ 3x + y + 4z + 11w &= 2, & 5x - 9y - 2z + 4w &= 7. \end{aligned} \quad [10]$$

4. Answer the following:

- (a) Use Runge-Kutta method of fourth order to find $y(0.1)$, (0.2) ;

$$\text{given that } \frac{dy}{dx} = y - x, \quad y(0) = 2. \quad [10]$$

OR

- (b) (i) Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$, find $\sin 52^\circ$. [5]

- (ii) Using Euler's method solve $\frac{dy}{dx} = x + y + xy$, at $x = 0.1$, given $y(0) = 1$, $h = 0.025$. [5]

5. Answer the following:

- (a) In a group of 2000 men, 5% are under 60 inches in height and 40% are between 60 and 65 inches. Assuming a normal distribution, find the mean height and standard deviation. [10]

OR

- (b) (i) In answering a question on a multiple choice test, an examinee either knows the answer (with probability p) or he guesses the answer (with probability $1-p$). Assume that the probability of answering a question correctly is unity for an examinee who knows the answer and $1/5$ for the examinee who guesses, where 5 is the number of multiple choice alternatives. Supposing an examinee answers a question correctly, What is the probability that he really knows the answer? [5]
- (ii) What is Binomial distribution? Derive mean of the Binomial distribution. [5]

6. Answer the following:

- (a) Fit an exponential curve of the form $Y = ab^x$ to the following data:

| | | | | | | | | | |
|----|-----|-----|-----|-----|-----|-----|-----|-----|------|
| X: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| Y: | 1.0 | 1.2 | 1.8 | 2.5 | 3.6 | 4.7 | 6.6 | 9.1 | [10] |

OR

- (b) Explain the method of fitting of exponential curves (i) $Y = ab^x$ and (ii) $Y = ae^{bx}$. [10]